

LINEAR PROGRAMMING PROBLEM

MATHEMATICAL FORMULATION

INTRODUCTION

Many business and economic situations are concerned with a problem of planning activity. In each case, there are limited resources at your disposal and your problem is to make such a use of these resources so as to yield the maximum production or to minimize the cost of production, or to give the maximum profit etc. Such problems are referred to as the problem of constrained optimization. Linear Programming is a technique for determining an optimum schedule of interdependent activities in view of the available resources. Programming is just another word for 'planning' and refers to the process of determining a particular plan of action from amongst several alternatives. The word linear stands for indicating that all relationships involved in a particular problem are linear.

MATHEMATICAL FORMULATION OF THE PROBLEM

The procedure for mathematical formulation of a linear programming problem consists of the following major steps:

- *Step 1.* Study the given situation to find the key decisions to be made.
- *Step 2.* Identify the variables involved and designate them by symbols x_j ($j=1,2,3,\dots$).
- *Step 3.* State the feasible alternatives which generally are : $x_j \geq 0$, for all j .
- *Step 4.* Identify the constraints in the problem and express them as linear inequalities or equations, LHS of which are linear functions of the decision variables.
- *Step 5.* Identify the objective function and express it as a linear function of the decision variables.

SAMPLE PROBLEM

1. A garment manufacturer has a production line making two styles of shirts. Style I requires 200 grams of cotton thread, 300 grams of dacron thread, and 300 grams of linen thread. Style II requires 200 grams of cotton thread, 200 grams of dacron thread and 100 grams of linen thread. The manufacturer makes a net profit of Rs. 19.50 on Style 1, Rs. 15.90 on Style II. He has in hand an inventory of 24 kg of cotton thread, 26 kg of dacron thread and 22 kg of linen thread. His immediate problem is to determine a production schedule, given the current inventory to make a maximum profit. Formulate the LPP model.

x_1 = Number of Style I shirts,

x_2 = Number of Style II shirts.

Since the objective is to maximize the profit, the objective function is given by —

$$\text{Maximize } Z = 19.50x_1 + 15.90x_2$$

Subject to constraints:

$$200x_1 + 200x_2 \leq 24,000 \quad [\text{Maximum Qty of Cotton thread available}]$$

$$300x_1 + 200x_2 \leq 26,000 \quad [\text{Maximum Qty. of Dacron thread available}]$$

$$300x_1 + 100x_2 \leq 22,000 \quad [\text{Maximum Qty. of Lines thread available}]$$

$$x_1, x_2 \geq 0 \quad [\text{Non-Negativity constraint}]$$